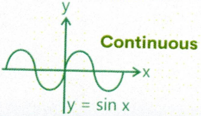
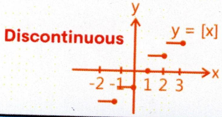
CONTINUITY & DIFFERENTIABILITY

CONTINUITY





Continuity at a point

If f(a) exists and $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

Some Continuous Functions

• **Identity** ; $f(x) = x$	• **Constant** ; $f(x) = c$
• **Polynomial** ; $f(x) = a^0x^n + a^1x^{n-1} + \ldots\ldots + a^n$	• **Trigonometric** ; $f(x) = \sin x, \cos x, \tan x$
• **Exponential** ; $f(x) = e^x, a^x$	• **Logarithmic**
• **Hyperbolic** ; $f(x) = \sinh x$	• **Absolute Value** ; $\lvert x\rvert$

- If g(x) is continuous at x = a and f(x) is continuous at g(a) then f(g(x)) is continuous at x = a.
- If g(x) is discontinuous at x = a then f(g(x)) may or may not be discontinuous at x = a.
- If g(x) is undefined at x = a then f(g(x)) is discontinuous at x = a.

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Continuity in an Interval

f(x) is said to be continuous in

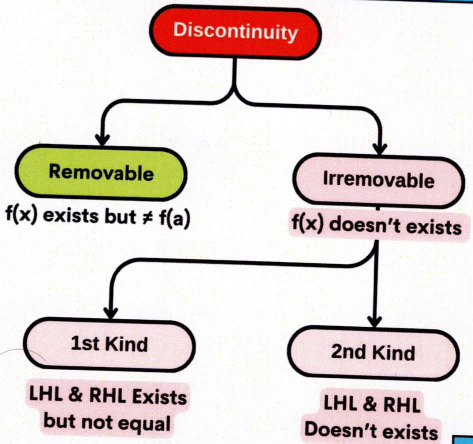
- **Open interval (a,b)** : when it is continuous at every point in (a, b).
- **Closed Interval [a,b]**

  a. Continuous $\forall x \in (a,b)$

  b. $\lim_{x \to a^-} f(x) = f(a)$ (Left hand Continuous)

  c. $\lim_{x \to a^+} f(x) = f(a)$ (Right hand Continuous)

Discontinuity



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Intermediate Value Theorem

If y = f(x) is continuous on [a, b] and k is any number between f(a) and f(b) then ∃ at least one number 'c' such that f(c) = k.

- If f(x) is continuous in [a, b] then there is at least one value of x say 'c such that $f(c) = \frac{f(a)+f(b)}{2}$
- If f(x) is continuous in [a, b] & f(a).f(b) < 0 then f(x) has at least one root in (a, b).

Differentiability

- The instantaneous rate of change of a function with respect to the dependent variable.

• Right Hand Derivative	• Left Hand Derivative
$f'(a^+) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$	$f'(a^-) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$

Provided that the above limits exists, f(x) is derivable or differentiable at x = a if, $f'(a^+) = f'(a^-)$ = finite value

- If a function is not differentiable but is continuous at a point, it geometrically implies there is a sharp corner or a kink at that point.
- If a function is differentiable at a point, then it is also continuous at that point.
- If a function is continuous at point x = a, then nothing can be guaranteed about the differentiability.
- If a function f(x) is not differentiable at x = a, then it may or may not be continuous at x = a

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- if a function f(x) is not continuous at x = a, then it is not differentiable at x = a
- If the left hand derivative and the right hand derivative of f(x) at x = a are finite (they may or may not be equal), then f(x) is continuous at x = a.

Important Points about Differentiability

Differentiability $\Rightarrow$ Continuity but Continuity $\nRightarrow$ Differentiability

All polynomial, trigonometric, logarithmic and exponential functions are continuous and differentiable in their domains.

- If f(x) & g(x) are differentiable at x = a then the functions f(x) + g(x), f(x) – g(x), f(x) g(x), f(x) / g(x) $(g(a) \neq 0)$ will also be differentiable at x = a

Differentiability in an interval

f(x) is said to be differential in

- **Open (a,b)** : when it is Derivable at every point in (a, b).
- **Closed [a,b]**

  a. Differentiable $\forall x \in (a,b)$

  b. Right hand derivative exists at x = a

  c. Left hand derivative exists at x = b

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